$$\text{Normal Vector} = \frac{d}{dt}\left[\frac{\text{Unit tangent Vector}}{\text{Magnitude of tangent Vector}}\right]$$

* Scalar And Vector Point functions :-

(i) field :-

If a function is defined in any region of space, for every point of the region, then this region is known as field.

(ii) Scalar Point function :-

A function $\phi(x, y, z)$ is called a scalar point function if it associates a scalar with every point in space.
The temperature distribution in a heated body, density of a body and potential due to gravity are the examples of a scalar point function.

(iii) Vector Point function :-

If a function $F(x, y, z)$ defines a vector at every point of a region, then $F(x, y, z)$ is called a vector point function. The velocity of a moving fluid, gravitational force are the example of vector point function.

$\vec{\nabla}$ is a vector Operator
Vetor $\neq$ Vector operator.

* Gradiant of a Scalar Field or Scalar Function :-

The vector differential Operator Del is denoted by ∇. It is defined as -

$$\underset{\text{grad / del operator}}{\vec{\nabla}} = \frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}$$

$\vec{\nabla}$ can not exist independentily.

$$\underbrace{\vec{\nabla}\cdot\vec{B}}_{\text{scalar quantity}} \neq \underbrace{\vec{B}\cdot\vec{\nabla}}_{\text{scalar operator}}$$

Lets consider a scalar function ϕ then

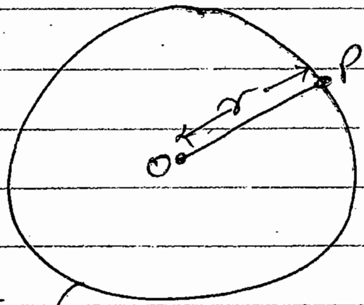
$$\vec{\nabla}\phi = \frac{\partial}{\partial x}\phi\,\hat{i} + \frac{\partial}{\partial y}\phi\,\hat{j} + \frac{\partial}{\partial z}\phi\,\hat{k}$$

$$\vec{\nabla}\phi = \underbrace{\frac{\partial \phi}{\partial n}}_{\text{magnitude.}}\ \underbrace{\hat{n}}_{\text{direction}}$$

⇒ Gradiant of a scalar function ϕ at any point $P(x,y,z)$ is a vector quantity whose magnitude is equal to the rate of change of ϕ with distance along to normal to the level surface and its direction is along normal to the level surface.

* **Level Surface :-** It is the surface on which the value of the scalar function ϕ is same at all points.

ex-



$$V = \frac{1}{4\pi\epsilon_0}\frac{q}{r}$$ (V is a scalar quantity)

Equipotential Surface is an example of level surface.

* **General Equation :-**

$$\phi = \text{Constant.}$$

⇒ Let equation of surface -

$$x^2+y^2+z^2 = r^2 \quad (\text{eq}^n \text{ of sphere})$$

define

$$\phi = x^2+y^2+z^2$$

$$\phi = \text{Constt.}$$

(2) $x^3y + yz^3 + yz = 8$

Corresponding ϕ define -

$$\phi(x,y,z) = x^3y + yz^3 + yz = \text{Const}$$

⇒ We have to define a scalar function ϕ which can be a function of (x, y, z).